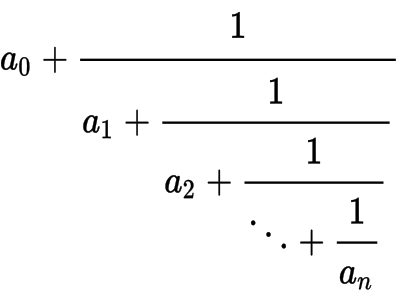
**What’s a continued fraction, and what is it good for?**

A continued fraction is a number represented through an iterative series of fractions of the form:

Using this form can show deeper patterns in the structure of numbers, and can be used for approximations of irrational numbers, often with small amounts of computation.

**Another way of writing continued fractions**

The more fractions that form a number, the more cumbersome they are to display. This problem leads to another form for writing continued fractions, as a list [a0;a1;a2; … an], with the elements in this particular list corresponding to the picture above.

**Evaluating a continued fraction**

To get the number to its simple form of a single numerator and denominator, evaluate the fraction at the very bottom, and work backwards, using the result of that fraction to evaluate the next one. For example:

**Forming a continued fraction:**

Take the rational number 123/49. This can be written as 2 + 25/49. Taking the reciprocal of 25/49 we get 49/25, which can be written as 1 + 24/25. The reciprocal of 24/25 being 25/24 which is 1 + 1/24. The reciprocal of 1/24 is simply 24, which terminates the continued fraction.

With irrational numbers take the first number of the resulting decimal, do the reciprocal of the remaining numbers and work from there.

All rational numbers will eventually terminate through this process and give a continued fraction which can be evaluated. However, irrational numbers will never terminate through this process, and instead give infinite continued fractions, which are incredibly useful for giving approximations to these numbers, with more iterations giving more accuracy. Some notable infinite continued fractions are shown below:

Pi: [3; 7; 15; 1; 292; 1; 1; 1; 2; 1; 3; 1; 14; 3 … ]

e: [2; 1; 2; 1; 1; 4; 1; 1; 6; 1; 1; 8; 1; 1; 10; 1; 1; 12; 1; 1; 11 … ]

sqrt(2): [1;2;2;2;2;2;2;2;2;2;2;2;2;2;2;2;2;2;2;2;2;2;2;2 … ]

The Golden ratio, phi: [1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1;1 … ]

**Rate of Approximations:**

Some approximations require more computation that others to converge to the desired result. Having larger values for denominators will cause the fraction to approach more quickly. Famous mathematician Ramanujan worked closely with infinite fractions and found remarkably accurate approximations. Take pi^4.

[a;b;c;d;e;16539 …. ]

Notice that the 6th element is abnormally large, which allows a continued fraction terminating with 16539 to give a very accurate pi^4. Taking the 4th root of this, a close approximation to pi is given (2143/22)^(1/4)

Now, looking at the golden ratio we have:

[1;1;1;1;1;1;1;1;1;1;1;1;1;1;;1;1;1… ]

…which has the smallest possible values for each denominator, meaning it takes many iterations to get close to the real value. For this reason, the golden ratio is often referred to as the “most irrational” fraction.

A table of steps and correct decimal points at each step? (comparing golden ratio to something like pi)

**References:**

https://www.youtube.com/watch?v=CaasbfdJdJg&ab\_channel=Mathologer  
https://plus.maths.org/content/chaos-numberland-secret-life-continued-fractions

**Image Sources:**

http://codegolf.stackexchange.com/questions/93223/simplify-a-continued-fraction

**(\*OCaml code to evaluate continued fraction given a list\*)**

let rec cftodec xs acc = match xs with

| [] -> 0.

| [x] -> x +. acc

| x::xs -> cftodec xs (1. /. (x +. acc));;

let float\_rev xs = List.rev\_map float xs;;

let cf2dec xs = cftodec (float\_rev xs) 0.;;

**(\*OCaml code to create a continued fraction from a decimal (requires previous code. Maximum value of 15 for n due to the accuracy of OCaml float)\*)**

let rec dectocf decimal acc originaldec accuracy =

if((abs\_float ((cf2dec (List.rev acc)) -. (originaldec))) <= accuracy) then List.rev acc

else

let intpart = truncate decimal in

let decpart = decimal -. (float intpart) in

dectocf (1./.decpart) (intpart::acc) originaldec accuracy;;

let rec dec2cf decimal n = dectocf decimal [] decimal (1./.(10. \*\* (float n)));;