**Intro**

Show patterns, good for approximations with minimal computation etc lmao

**How to calculate continued fractions of real numbers:**

Continued fractions are built through an iterative process in which an element is added to a fraction of 1 over the rest of the number (as shown in figure 1). This can be cumbersome to display, as the more steps there are in a continued fraction, the more cluttered it will get when writing it out. This leads to another format for writing continued fractions, as a list [a0;a1;a2;a3;a4 … an] for n elements.

**Evaluating a continued fraction:**

To get the number to its simple form of a single numerator and denominator, evaluate the fraction at the very bottom, and work backwards, using the result of that fraction to evaluate the next one. For example:

**Forming a continued fraction:**

Take the rational number 123/49. This can be written as 2 + 25/49. Taking the reciprocal of 25/49 we get 49/25, which can be written as 1 + 24/25. The reciprocal of 24/25 being 25/24 which is 1 + 1/24. The reciprocal of 1/24 is simply 24, which terminates the continued fraction.

With irrational numbers take the first number of the resulting decimal, do the reciprocal of the remaining numbers and work from there.

All rational numbers will eventually terminate through this process and give a continued fraction which can be evaluated. However, irrational numbers will never terminate through this process, and instead give infinite continued fractions, which are incredibly useful for giving approximations to these numbers, with more iterations giving more accuracy. Some notable infinite continued fractions are shown below:

Pi:

e:

sqrt(2):

The Golden ratio, phi:

**Rate of Approximations:**

Some approximations require more computation that others to converge to the desired result. Having larger values for denominators will cause the fraction to approach more quickly. Famous mathematician Ramanujan worked closely with infinite fractions and found remarkably accurate approximations. Take pi^4.

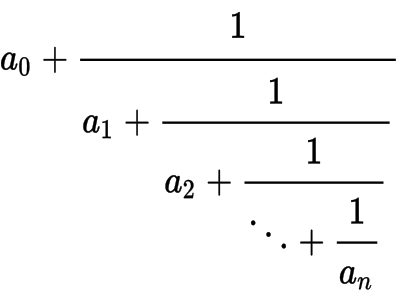
[a;b;c;d;e;16539 …. ]

Notice that the 6th element is abnormally large, which allows a continued fraction terminating with 16539 to give a very accurate pi^4. Taking the 4th root of this, a close approximation to pi is given (2143/22)^(1/4)

Now, looking at the golden ratio we have:

[1;1;1;1;1;1;1;1;1 …… ]

…which has the smallest possible values for each denominator, meaning it takes many iterations to get close to the real value. For this reason, the golden ratio is often referred to as the “most irrational” fraction.

A table of steps and correct decimal points at each step? (comparing golden ratio to something like pi)

**References:**

<https://www.youtube.com/watch?v=CaasbfdJdJg&ab_channel=Mathologer>  
<https://plus.maths.org/content/chaos-numberland-secret-life-continued-fractions>

**Image Sources:**

<http://codegolf.stackexchange.com/questions/93223/simplify-a-continued-fraction>